Alarming Realities:

- “Mathematics, more than any other subject, has the power to crush students’ confidence (4/10)” (pg. xvii).
- “More than half of all U.S. students fail mathematics” (pg. 3).
- “70% of students attending a two-year college are placed in remedial math classes, repeating the math they took in high school” (pg. 4).
- “In 64 countries across the world, the U.S. ranked 36th in mathematical performance. When levels of spending in education were taken into account, the U.S. dropped to the bottom of all the tested countries (pg. 3).
- “For, about 15 million students in the U.S., math ends their college career” (pg. 4).
The Brain and Mathematics Learning:

- **Brain Plasticity (Flexibility):**
  - Brains can grow and change within a really short period (10 min. activity for three weeks).
  - “If you learn something deeply, the synaptic activity will create lasting connections in your brain, forming structural pathways, but if you visit an idea only once or in a superficial way, the synaptic connections can ‘wash away’ like pathways made in sand” (Boaler).

- There is no such thing as a “math brain” or a “math gift.” This is a fixed mindset (opinion).
  - Everyone can learn math well.
  - The brain is malleable (working memory, hippocampus, IQ-all based on life experiences/hard work and determination)
  - “Research is showing students are better problem-solvers BEFORE attending math class” (pg. 43).
Fixed vs. Growth Mindset

- Students (fixed mindset) are more likely to give up easily.

- Students (growth mindset) keep going (persistent) when work is hard, displaying “grit” (highest achieving students in the world). They view mistakes as a challenge (intrinsic motivation).

- Mindsets can change from fixed to growth.

- Fixed mindsets can be damaging (believing/told you are smart).

- Praise parents gave babies (birth to three) led to predictable mindsets five years later (children).

- Students should be praised for effort vs. fixed praise.

- Two paths: success/enjoyment, or frustration/failure
The Power of Mistakes and Struggle:

- **Mistakes are a good thing!**
  - “Every time a student makes a mistake in math, they grow a synapse” (Boaler).
  - The brain has two possible responses when we make a mistake:
    a. Increased electrical activity (ERN response) whether we recognize the mistake or not.
    b. A brain signal reflecting conscious attention to mistakes (Pe response)
  - Why? It is a time of struggle where the brain is challenged, and this is the time the brain grows the most.
  - Those with a growth mindset are more aware of mistakes than those with a fixed mindset. Also, have greater brain activity related to error recognition.
How can we change the ways students/teachers view mistakes?

• Embrace and celebrate mistakes. (Inform parents)

• Make mistake celebrations a classroom norm.
  - Feel comfortable being wrong
  - Try seemingly wild ideas
  - Be open to different experiences (student posters with brain messages)
  - Play with ideas without judging them
  - Be willing to go against traditional ideas
  - Keep going through difficulties (crumpled paper activity)
  - Abandon testing and grading as much as possible (give the same grade or higher for mistakes)

Ø Piaget: True wisdom = moving from equilibrium, to disequilibrium, to a new state of equilibrium
Taming the Monster (Testing):

• American children are tested more than ever, more than the rest of the world, and the tests are rejected by most countries (pg. 84).

• What we can control (Assessment for learning): creating self-regulating learners who have the knowledge and power to monitor their own learning.

• Negatives of Multiple Choice Tests:
  a. Graded by machines (0.932 math=language tests)
  b. Not supported anywhere else (does not assess understanding, biased for ethnic minority students, causes anxiety (timed), does not give you accurate information, too narrow (procedures))
  c. Have a huge/damaging impact on what is taught in schools (gaps, grades, stereotypes, labels).
Assessment for Learning (pgs. 95-102):

Students should have a full clear sense of what they are learning (designed to promote learning):

1. Communication about what is being learned and where students are going (traffic lighting).

2. Making individual students aware of where they are on the path to success.

3. Giving students clear advice about how to become more successful (diagnostic feedback, not grades).

4. Includes self- and peer-assessments: Students are often better able to hear criticism from their peers than from a teacher, and peers usually communicate in easily understood ways (two stars and a wish) (pg. 97).
The Creativity/Beauty in Mathematics:

• Mathematics is different from other subject areas:
  Ø Due to false fixed beliefs (performance subject, “speed” = “high/gifted in math)
  Ø The testing culture is a large part of the problem.
  Ø Wide difference between real mathematics and school mathematics (incredibly misrepresented)
  Ø Only one of the four stages of mathematics is being used in the classroom.
  Ø Most valued skills in society have changed (Fortune 500)

• What is “real mathematics?”
  Ø “It is a subject full of exploration, conjectures, and interpretations, not definite answers… at its core, mathematics is about patterns” (Boaler).
  Ø “Mathematics is a human activity, a social phenomenon, a set of methods used to help illuminate the world (relationships)” (Boaler).
How Should We Prepare Our Students?

For students to TRULY understand mathematics:

- We need to “teach students to be quantitatively literate, think flexibility and creatively, problem solve, and use intuition as they develop mathematical ideas” (pg. 8).
- “Children need to solve complex problems, to ask many forms of questions, and to use, adapt, and apply standard methods as well as to make connections between methods and to reason mathematically” (pg. 11).
- Learn to estimate and make guesses, mathematics is not all about precision (do not emphasize memorization) (pg. 25).
- Encourage collaboration, not isolation with math (pg. 26). To truly understand, students need to explain their thinking (reason and justify) (pgs. 46-48).
- Promote a range of representations (language, symbols, words, pictures, tables, and diagrams) (pg. 28).
- Give them time to set up their equations (pg. 45).
Views on Math:

• **Four Stages of Mathematics:**
  - Posing a question (data/situation)
  - Going from the real-world to a mathematical model (to answer the question)
  - Performing a calculation (80% of school mathematics is spent here. It is the one stage employers do not need workers to do.)
  - Going from the model back to the real-world, to see if the original question was answered

• **What do employers need?**
  - People to think and reason
  - People who can ask good questions, and set up models.
  - People who can analyze results, and interpret mathematical answers.
The Importance of Flexibility with Numbers (Number Sense):

- Why does the love of mathematics change from infants to childhood?
  - It goes from playing with numbers and shapes, and thinking about patterns and ideas they can see and learn into systematic tests on simple ideas with hundreds of methods and procedures to memorize that they will never use, and hundreds of answers to questions that THEY have never asked. It is not relevant/meaningful.

- Number sense is the foundation for all higher-level mathematics (not rules, but processes).
  - Downward spiral for “struggling students.”
  - Compression process is critical.
  - We need to teach math facts differently.
  - We need to change our questions/roles.
Creating Mathematical Mindsets:

- In Regards to Math and Math Facts:
  - Students can learn and memorize math facts through conceptual engagement, not mindless practice and speed drills (1/3 experience anxiety with timed tests).
  - Learn through strategies.
  - Stop endless practice problems! (Induces boredom and it gives the most simplified and disconnected version of the method to be practiced)
  - Stop with “perfect examples,” and include “non-examples.”
  - Stop mundane homework (effective when a worthwhile learning experience/meaningful task, not practice problems on worksheets). What life changing idea did you learn from a worksheet?
  - Make it engaging! Games/Apps.
  - “Mathematical notation no more is mathematics than musical notation is music” (pg. 29).
Rich Mathematical Tasks:

• Teachers are the most important resource for students!
  ➢ Teachers can create exciting mathematical environments, give students the positive messages they need, and take any math task integrate curiosity and interest. The curriculum can be a teacher’s best friend.
  ➢ Promote mathematics excitement: combines curiosity, connection making, challenge, creativity, and collaboration
  ➢ Give important messages/open up mathematical tasks
  ➢ Promote equity (heterogeneous groups)
  ➢ Promote real examples, not Mathland! (pg. 51)
  ➢ Encourage/Celebrate mistakes
  ➢ Allow everyone to experience high level math!
Mathematics and the Path to Equity:

• **The Elitist Construction of Math:**
  - The myth of the mathematically gifted child (there is no “guardian of math”)
  - When math inequalities in course placement become illegal (break the cycle of low achievement)

• **Equitable Strategies:**
  - Offer all students high-level content.
  - Work to change the ideas about who can achieve in mathematics.
  - Encourage students to think deeply about mathematics (hands-on, project-based, curriculum with real-life applications, opportunities to work together).
  - Teach students to work together (relational equity).
  - Give girls and students of color additional encouragement to learn math and science (research).
  - Eliminate homework (or, at least, change it).
What We Need to Do/Stop Doing:

• Stop with the labels (gifted, low, etc.)!
  - These are examples of fixed mindsets influencing/hindering futures of our students.
  - “Math placement that results in inequalities is a legal offense” (Boaler).
  - True mathematics teachers (K-12), “teach heterogeneous math classes using complex instruction, enabling all students to take high-level math classes” (Boaler).

• Balancing Aspects of Mathematics:
  - Conceptual understanding
  - Problem-solving
  - Computational/Procedural skills (Be careful with these.)
  - “Learning is...a process of identity development, as students decide who they are and want to be” (Boaler).
Growth Mindset Grouping (No Tracking):

• **What to Give Students:**
  - Opportunities to learn
  - De-Tracking (Reduces achievement-fixed mindset): Low floor, high ceiling tasks (reject ability grouping)
  - Tracking/ability groups is/are illegal in many countries.
  - Growth Mindset Grouping (bad behavior and lack of motivation disappear, give choices for classes)

• **Teaching Heterogeneous Groups Effectively:**
  - Providing open-ended tasks
  - Offering a choice of tasks
  - Individualized pathways
  - Complex Instruction: multidimensionality (all ways to be mathematical), roles, assigning competence, teaching students to be responsible for each other’s learning)
Tracking = Stuck in the Slow Lane:

- Negatives of Ability Grouping:
  - Tracking/ability groups is/are illegal in many countries (Japan, Finland, Korea).
  - We are not identifying students correctly! Children develop at different rates due to unequal schooling, and we are basing it off of assessments that do not measure student learning. “Everyone can do everything; we think that is a good thing, so we can’t divide by ability” (student perspective in Japan, pg. 105)
  - The students know, and will give up. Even “high-students” felt pressured and inadequate.
  - Teachers routinely underestimate children in low groups (even if unaware).
  - Mixed ability groups = higher achievement levels of ALL students.
  - Tracking leads to damaging fixed mindsets.
Tracking = Stuck in the Slow Lane (pt. 2):

Other Negatives Regarding Ability Grouping:

- Students become fearful of making mistakes, and start to avoid challenging work (pg. 110).
- Ability groups tend to focus on students in the middle group. **Majority of the work is at the wrong pace or the wrong level for the rest of the groups.**
- When teachers assign students to different ability groups, they make decisions that affect their students’ long-term achievement and their **life chances** (one missed point, scored on another day, ends up limiting that student’s achievement for the rest of their lives) (pg. 111).
- Ability groups damage oral language development, student respect, student resources, and are viewed as “psychological prisons” (pgs. 117-119).
- “88% of children placed into ability groups at age 4 remain in the same groupings until they leave school” (pg. 119).
What’s a Multidimensional Classroom?

• In a multidimensional math class, teachers:
  ➢ Value and assess students on the many different dimensions of math. “No one is good at all of these ways of working, but everyone is good at some of them” (Boaler).
  ➢ Involve: asking good questions, rephrasing problems, explaining, using logic, justifying methods, using manipulatives, connecting ideas, and helping others

• Example?
  ➢ “Encourage students to read the problems out loud, and when they are stuck, let them ask each other questions (What is the question asking us? How could we rephrase the question? What are the key parts of the problem?)” (Boaler).
Student Engagement:

- **Student engagement is due to many factors.**
  - The teacher carefully sets up the problem and circulates around the room asking students question (make your desk temporarily inaccessible).
  - The task needs to be sufficiently open and challenging to allow students to contribute ideas.
  - It offers different ways to work mathematically (asking questions, drawing diagrams, making conjectures that are valued and encouraged).
  - We should work with real-world objects and ideas (solve REAL problems: number talks).
  - There needs to be a high level of communication among students helps them to learn to support each other by asking questions (to each other).
  - Supporting both genders (ways of learning) in the classroom (why and how vs. memorization).
Assigning Competence and Responsibility:

• What does this involve?
  ➢ Assigning competence involves raising the status of students who they (the teacher) think may be of a lower status in a group. Example: Praising something the student has said or done that has intellectual value, and bringing it to the group’s or whole class’s attention.
  ➢ Responsibility involves reminding students of the classroom norms that the teacher and students agreed upon. The students learn how to work well together (engaging in respectful conversation, listening to each other, and building upon each other’s ideas (increases relational equity)).
  • Example: Always give help when needed, always ask for help when you need it.
  • Increase the promotion of equity in all subject areas.
Assessment for a Growth Mindset:

• What is good testing?
  - It should assess what is important.
  - “It should measure student learning”
  - It should be an assessment.
    Formative: Informs where the students are in their learning, to determine what they need to know next.
    Summative: summary, end of unit/concept

• In a Nutshell:
  - Stop any tests you have control over (unit test).
  - Stop grades (Race to Nowhere, and Beyond Measure).
  - Instead, clearly communicate where students are now, where students need to be, and ways to close the gap.
Developing Strong Math Characteristics:

• Developing Student Self-Awareness and Responsibility:
  ➢ “The most powerful learners are those who are reflective, who engage in metacognition (thinking about what they know) and who take control of their own learning” (pg. 150).

• Ways to Establish These Characteristics:
  ➢ Self assessments (clear statements of the math they are learning) and Peer assessments (clear criteria to assess each other’s work: two stars and a wish)
  ➢ Reflection time (during class time)
  ➢ Traffic Lighting (red = stop, yellow = slow down, green = keep going), Jigsaw groups (work together to become experts), Exit tickets, Online forms, Doodling (to show understanding), student write questions and tests (for other students).
Comments and Advice:

• Diagnostic Comments:
  ➢ “Students need to understand how close the gap between where they are and where they need to be… giving your knowledge, ideas, and feedback on their mathematical development, when phrased positively and with growth messages,” is the greatest gift a teacher can give a student (Boaler).

• Advice on Grading:
  ➢ Always allow students to resubmit any work/test for a higher grade.
  ➢ Share with school administrators, not with students. Use diagnostic feedback with students instead.
  ➢ Use multidimensional grading
  ➢ Do not use a 100-point scale (instead, 4-point scale).
  ➢ Do not include early assignments from math class in the end-of-class grade.
  ➢ Do not include homework, as any part of grading.
Teaching Mathematics for a Growth Mindset:

• Encourage all students.
  ➢ Set up positive classroom norms.
  ➢ Participation norms.

• Believe in all of your students.

• Value struggle and failure.

• Give growth, praise, and help.

• Opening mathematics.
Growth Mindset Model 1: The Communicative Approach (pg. 58)

- Teachers collaborate and plan teaching ideas together. “Careful about identifying and talking to students about all the ways they were smart” (pg. 68).

- The curriculum is organized around bigger mathematical ideas with unifying themes, and multiple representations.

- Views/promotes math as a form of communication, or a language (open problems).

- Organized in heterogeneous groups that help each other (discussion based).

**End result:** Students learn to enjoy math!
Growth Mindset Model 2: The Project-Based Approach (pg. 68)

• The teachers give the students projects to work on that need mathematical methods (less order and control). Work on open-ended projects in every lesson. **End result:** Students enjoy math!

• Introduce students to a problem/theme that the students explored, using their own ideas and mathematical methods that they are learning (Sometimes, teachers show students mathematical concepts that would be useful before the start of a new project.).

• Students are given choices between projects (suited to their strengths) to work on, and the direction of their work (freedom). Given choice on when to work in groups, pairs, or alone.
Opening Mathematics:

- Teach mathematics as an open, growing, learning subject (space for learning, exploring, creating).
- Encourage students to be mathematicians (Mathematicians view math as beautiful and aesthetic.).
- Teach mathematics as a subject of patterns and connections (active role in pattern seeking in all areas and in all levels of math).
- Teach creative and visual mathematics (the use of different brain pathways).
- Encourage intuition and freedom of thought.
- Value depth over speed (speed is a false impression).
- Connect mathematics to the world using mathematical modeling (offers more simplicity/relevancy).
- Model with mathematics (rich questions, investigate/data).
- Encourage students to pose questions, reason, justify, and be skeptical (tends to be absent in the classroom).
- Teach with cool technology and manipulatives.
Predictions of Student Achievement:

1. Counting All vs. Counting On Strategy
2. Known Facts
3. Derived Facts

Realizations:
1. High achieving students don’t “just know more.” Instead they work in a different way. “They engage in flexible thinking when they work with numbers, decomposing and recomposing numbers” (pg. 141).
2. Low achievers are often thought of as slow learners, when in fact they are not learning the same things slowly. They are learning a different mathematics.
3. The mathematics that low achievers are learning is a more difficult subject (large cognitive complexity and the room for mistakes is huge).
4. If a method failed, they did not change their method. They would try it again (Need to compress ideas!).
Processing Differences:

Low Achievers
Show students how to use numbers flexibly, and give them time to play with numbers!

High Achievers
Suggested Strategies (pgs. 172-195):

1. Nurture the motivation for students to come up with their own ideas.

2. Encourage mathematical thinking by giving interesting puzzles and problems to work on (think/reason).

3. Promote and asking questions. “What do you think you should do? How did you get that?”

4. Never say a child’s answer is wrong. Show them how their thinking was correct so that can learn about the ways it may be improved.

5. Be as enthusiastic as possible, or fake it until you believe it.
Suggested Strategies (continued):

6. Steer children away from the mathematical ladder of rules (Example: Number Talks).

7. Never praise children by telling them they are smart, praise WHAT the child has done.

8. Never share stories of math failure of even a dislike in math.

9. Always praise mistakes and say that you are really please that your student is making them.

10. Encourage children to work on problems that are challenging (productive struggle)

11. When you help students, do not lead them through the work step-by-step.
Suggested Strategies (continued):

12. Encourage drawing whenever you can.

13. Encourage students to make sense of mathematics at all times (Does that make sense to you? Why, or why not?)

14. Encourage students to think flexibly about numbers (biggest difference between successful students and not).

15. Never time children or encourage faster work (Don’t use flashcards or timed tests.).

16. When children answer questions and get them wrong, try to find logic in their answers-as they have usually used some logical thinking.
Suggested Strategies (continued):

17. Play games, which are similarly helpful for children’s mathematical development.

18. Use assessments, not tests, grades, or meaningless homework.

“Let us move together from the mathematics trauma and dislike that has pervaded our society...to a brighter mathematical future for all, charged with excitement, engagement, and learning. Viva la Revolution!” (Boaler, pg. 195)
Thinking Conceptually Questions:

1. What is the question asking you?
2. How could you draw this situation?
3. How did you get that answer? (Right or Wrong)
4. Can you share your method with me?
5. Can you try a different way of solving this?
6. What does (insert concept here) mean?
7. In what other situation could we use this?
8. Would this method work with different numbers?
9. What is important about this work?
Additional Information:

- Resources: Websites throughout this book, Appendix A-C
- This author, also, wrote the book, **Mathematical Mindsets**
- Large supporter of the book, **Making Number Talks Matter** by Cathy Humphreys and Ruth Parker
- “If we give students this rich, creative, growth mathematics experience, then we change them as people and the ways they interact with the world.” (Boaler).
- We need to purposefully make math more inclusive, for everyone (Boaler).
- When students become experts (responsibility of teaching peers), they are again encouraged to take responsibility for their learning (Boaler).
- “Our perceptions influence our students. Let’s stop saying, ‘I know this is really hard, or I was bad at math, or I didn’t like math.’ Instead, let’s fully embrace change, celebrate mistakes, and focus on developing our growth and mathematical mindsets (Boaler).